MAS212 Assignment 3: Extrasolar planets

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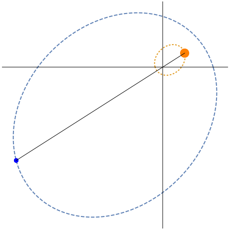
(a) Introduction to extrasolar planets



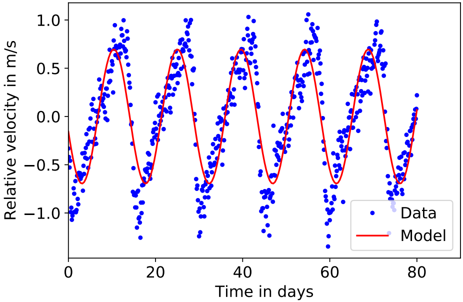
Are we alone in the Universe?

An extrasolar planet, or exoplanet, is a planet in orbit around a star other than the Sun.

Over **3700** exoplanets have been found;
but only a few ($\sim$ 15–45) are on orbits within the **habitable zone**, where liquid water may be present.

- **Instruments:** Kepler space obs. (2009-13); TESS; GPI; HARPS.
- **Methods:** 14 different methods used, such as transit photometry.
- **Radial velocity method:** Emission lines in the spectrum of the star shift in frequency, periodically, due to a Doppler shift caused by the star's motion in orbit with the planet.
- **Nearby planets** in the habitable zone: Ross 128b, the TRAPPIST planets and Proxima B ($\sim$ 4.6 light years away orbiting a red dwarf in Proxima Centauri).

(b) Fitting a linear model



- The linear model:
  $v(t) = f_0(t; \beta_0, \beta_1)$,

  $$f_0 = \beta_0 \sin(\omega t) + \beta_1 \cos(\omega t)$$

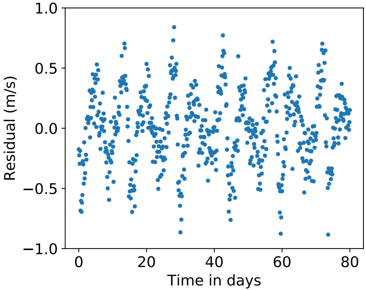
  where β_i are parameters and $\omega = 2\pi/14.6$.

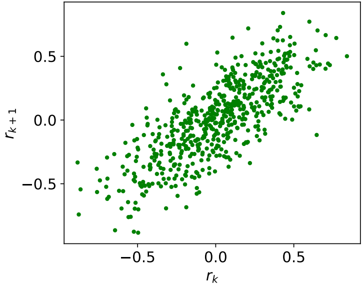
- Parameters β_i found from **normal equations**:

$$\left(\mathbf{X}^T\mathbf{X}\right)\boldsymbol{\beta} = \mathbf{X}^T\mathbf{y}$$

- **Best-fit parameters**: $\beta_0 = -0.678$, $\beta_1 = -0.157$ (to 3dp).

(c) Testing the goodness of fit





- *Upper:* Residuals after fitting:
  $$r_k = v_k - f_0(t_k; \beta_0, \beta_1).$$
  Lower: Highlighting the correlation between r_k and r_{k+1}.
- Root Mean Square Deviation:
  $$\text{RMSD} = 0.312 \quad \text{(3dp)}.$$
- Auto-correlation coefficient:
  $$\alpha_1 = 0.765 \quad \text{(3dp)}.$$
- The residuals are not 'random': there is correlation between successive residuals.
- A better model can be found.

(d) A physical model

- The Doppler shift is caused by the motion of the star, in an elliptical orbit with the extrasolar planet.
- The relative velocity of the star is $v_{\parallel} = s\cos\phi - r\dot{\phi}\sin\phi$ where

$$\dot{r} = s, \qquad r(0) = \frac{p}{1+e}, \qquad p : \text{semi-latus rectum},$$

$$\dot{s} = -\frac{1}{r^2} + \frac{p}{r^3}, \qquad s(0) = 0, \qquad e : \text{eccentricity},$$

$$\dot{\phi} = \frac{\sqrt{p}}{r^2}, \qquad \phi(0) = \phi_0, \qquad \phi_0 : \text{orientation}.$$

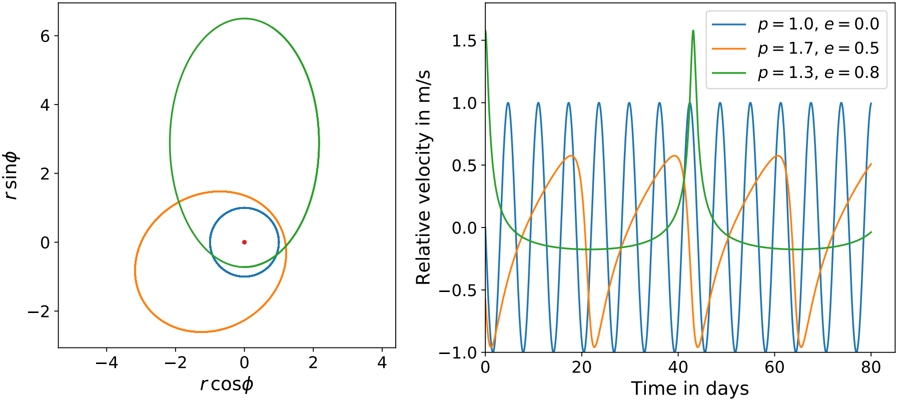
- I solved the 3D autonomous ODE system $\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X})$, where $\mathbf{X}(t) = (r, s, \phi)^T$, using the **midpoint method**:

$$\mathbf{X}_{k+1/2} = \mathbf{X}_k + \frac{1}{2}h\mathbf{F}(\mathbf{X}_k),$$

$$\mathbf{X}_{k+1} = \mathbf{X}_k + h\mathbf{F}(\mathbf{X}_{k+1/2}), \qquad t_{k+1} = t_k + h.$$

- The method is 2nd-order-accurate, i.e. global truncation error $O(h^2)$.

(e) Orbits and signals



- *Left:* Elliptical orbits. *Right:* Relative velocity of the star vs time.
- Close orbit ⇒ short period; Eccentric orbit ⇒ skewed profile.

(f) Fitting the physical model

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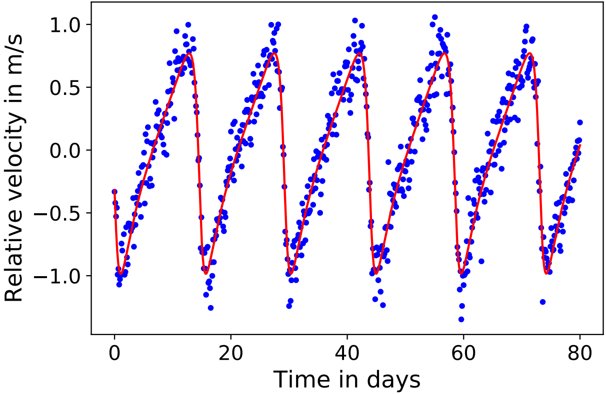

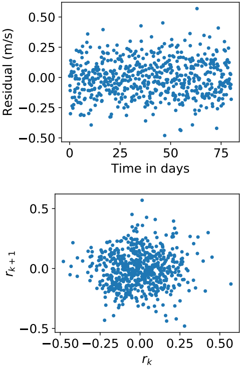

- RMSD = 0.152 (cf. 0.312 for linear model)
- No auto-correlation in residuals
- **Best-fit parameters**: $p = 1.299$, $e = 0.510$, $\phi_0 = 0.244$.

(g) Physical parameters

- **Kepler's 3rd law:** $a^3 = \frac{GM}{\omega^2}$, where $a = a_1 + a_2$, $M = m_1 + m_2$ (1=star, 2=planet).
- Here $m_1 = M_\odot \approx 2 \times 10^{30}\text{kg}$ and $m_1 \gg m_2 \Rightarrow a_2 \gg a_1$.
- **Semi-major axis:** $a = 1.752 \times 10^{10}\text{m} = 0.117\text{a.u.}$
  8.5 times smaller than the Earth.
- **Periastron** (closest approach): $r_p = a(1-e) \approx 0.057\text{a.u.}$
  5.35 times smaller than Mercury.
- **Planet's mass:** Following the derivation in the 2016 report, with $v_\parallel^{\max} \approx 0.8\text{ms}^{-1}$,

$$m_2 = v_\parallel^{\max} \sqrt{\frac{m_1 a_2}{G} \frac{(1-e)}{(1+e)}} \approx 1.04 \times 10^{25}\text{kg}.$$

  1.75 times the mass of the Earth.
- **Conclusion:** A 'rocky' planet like the Earth, but unlikely to support life as it passes ~ 17.5 times closer to its star.