MAS212 The Mandelbrot set

1. Your friend is making a poster for an Outreach event. They have asked you to make a high-resolution (500 × 500) image of the Mandelbrot set.

The Mandelbrot set is the set of complex numbers \( c \) for which the function \( f_c(z) = z^2 + c \) does not diverge when repeatedly iterated from \( z = 0 \).

Here is the Mandelbrot set on the complex domain \(-1.5 \leq \text{Re}(c) \leq 0.5, -1 \leq \text{Im}(c) \leq 1\):

2. You can use numpy arrays, together with universal functions (ufuncs), vectorization and broadcasting to help solve this problem quite efficiently.

   (a) Use broadcasting to make a 2D array \( c \) out of two 1D arrays made with np.linspace(). The array \( c \) should have data type dtype = np.complex128. The array \( c \) should be evenly spaced across the domain \(-1.5 \leq \text{Re}(c) \leq 0.5, -1 \leq \text{Im}(c) \leq 1\).

   (b) Make a 2D array \( z \) of the same shape, initially set to zero values.

   (c) Apply 100 iterations of the rule \( z \rightarrow z^2 + c \). Use vectorization/ufuncs.

   (d) Create a 2D boolean array, with value True if \( |z_{ij}| \leq z_{\text{max}} \), where \( z_{ij} \) is the corresponding element of the \( z \) array, and False otherwise.

   (e) Plot an image of the 2D array, using matplotlib.pyplot.imshow(). Save and send to your friend.
import numpy as np
import matplotlib.pyplot as plt

# Part (a).
xmin = -1.5; xmax = 0.5; ymin = -1; ymax = 1.0; # the domain
npts = 501; # number of points on each axis
xs = np.linspace(xmin, xmax, npts) # real parts
ys = np.linspace(ymin, ymax, npts, dtype=np.complex128)*1j # imag parts
# Now make a 2D array by broadcasting two 1D arrays
cs = xs.reshape((1, npts)) + ys.reshape((npts, 1))

# Part (b)
zs = np.zeros((npts, npts), dtype=np.complex128)

# Part (c).
nits = 100
for i in range(nits):
    zs = zs**2 + cs # using a ufunc

# Part (d).
maxval = 100.0
mandelbrot = np.abs(zs) < maxval

# Part (e)
fig = plt.imshow(mandelbrot, cmap='Purples', origin='lower')
plt.axis('off')
plt.savefig("mandelbrot.png")