MAS212 Assignment #3 (2019): Gravitational waves

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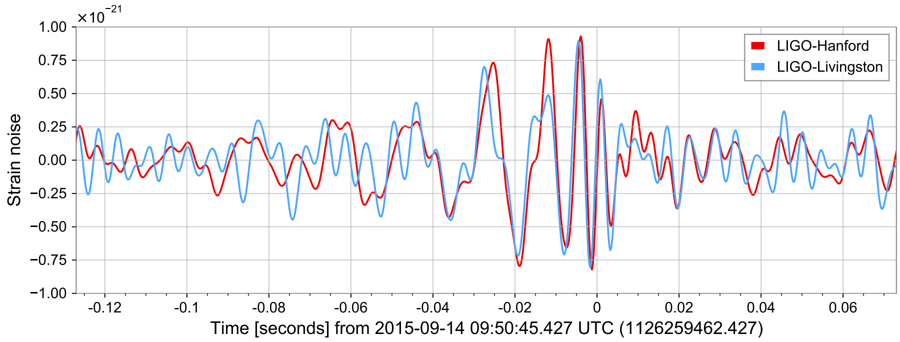
Gravitational waves are 'ripples in spacetime' that propagate at the speed of light. They are a prediction of Einstein's theory of general relativity (1915). Gravitational waves were first detected on Earth in 2015 at the Laser Interferometer Gravitational-wave Observatory (LIGO). Since then, more than 10 **chirps** from binary systems have been detected. A chirp is a signal which increases in frequency and amplitude in a particular way, before cutting off quite abruptly. Below is some real data seen in the two detectors of the LIGO experiment, showing a chirp in a noisy data channel [1]:



A chirp is the characteristic signal from a **compact binary** in its final stages of an inspiral orbit. A compact binary consists of a pair of black holes (or a pair of neutron stars; or one black hole and one neutron star) orbiting around each other. The binary emits gravitational waves at approximately twice the orbital frequency. Consequently, the orbit tightens, and the bodies spiral closer together. The frequency and the amplitude of the waves increases, until such a time as the bodies collide or merge.

An astronomy student has found a "chirp" in data from a gravitational-wave detector, and she needs your help to calculate the so-called **chirp mass** M of the binary system.

In this assignment you will:

- Learn a little about gravitational waves from binary systems.
- Fit a linear model to a data set;
- Examine some real data from the LIGO experiment.
- Prepare a **Beamer presentation**;

[1] You can make this plot for yourself using the GWpy package: `https://gwpy.github.io/docs/stable/overview.html`

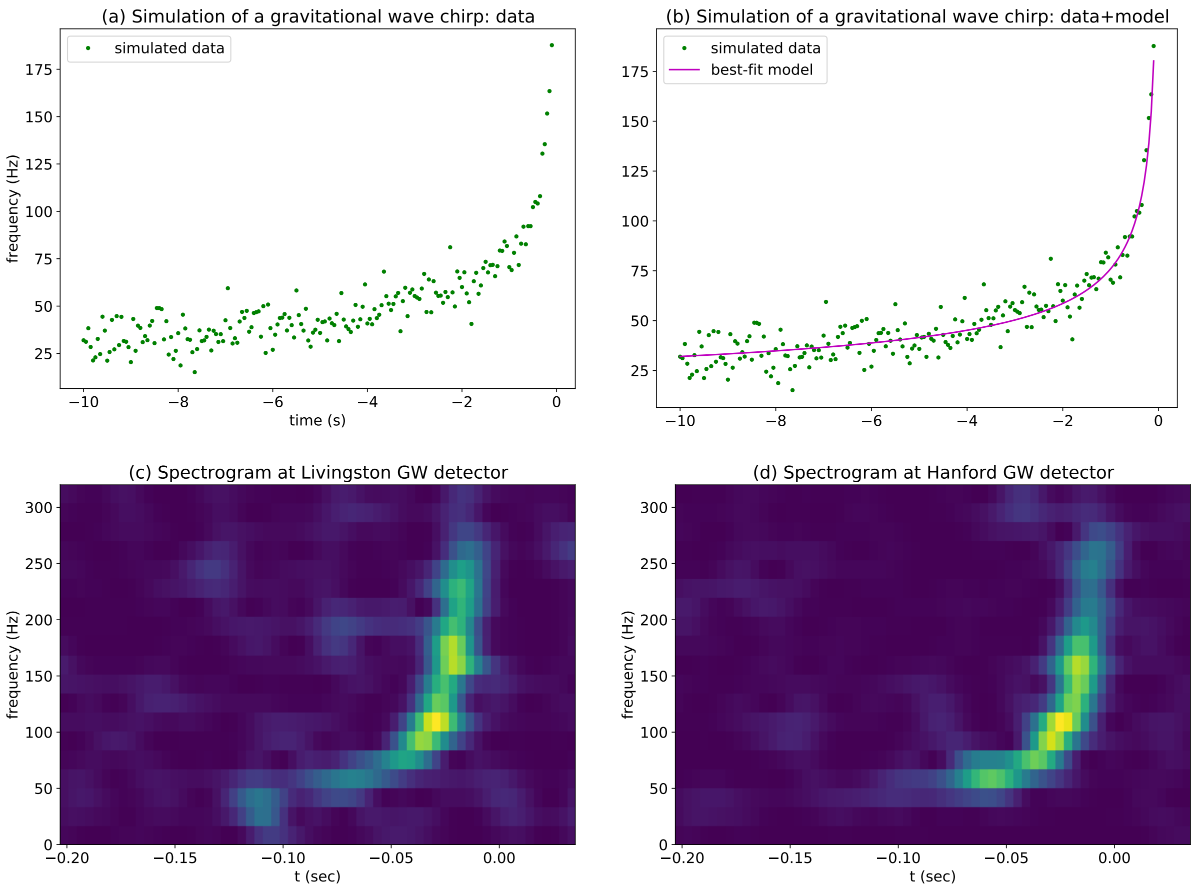


Figure 1: Gravitational wave chirps.
(a) top left: A simulation of data from a compact binary, in the time-frequency domain.
(b) top right: The model in Eq. (1) fitted to the data in (a).
Lower: Spectrograms made with real data from the Laser Interferometer Gravitational-wave Observatory.
(c) Hanford detector. (d) Livingston detector.

The Submission: A completed assignment will comprise a `.pdf` of your Beamer presentation and a single code file (`.py` or `.ipynb`). Your presentation will consist of the **slides** described below. Note that the presentation will carry more credit than the accompanying code. (N.B. *You will **not** be asked to give an oral presentation for this assignment*).

The Deadline: The deadline for submission is found on the course website. Files should be submitted at `https://somas-uploads.shef.ac.uk/mas212`.

The slides

- **Slide 0:** Registration number and presentation title only.
- **Slide 1:** Introduction.
- **Slide 2:** The signal.
- **Slide 3:** The model.
- **Slide 4:** Fitting a model: theory.
- **Slide 5:** Fitting a model: result.
- **Slide 6a:** The chirp mass or,
- **Slide 6b:** Analysing LIGO data.
- (optional) References and sources.

The steps

(0) Title & name. The first slide consists of only your registration number and the title of your presentation.

(1) Introduction. Read around the subject of gravitational waves. Prepare **one slide** on an introduction to gravitational waves to a Level 2 mathematics student. Keep it brief. No code is necessary here. If you include an image or picture, you must cite its source.

(2) The signal. The astronomer asks you to download and plot the data file here:
`http://sam-dolan.staff.shef.ac.uk/mas212/data/gw/`.
Use the data file whose filename is your registration number.
Plot the frequency f (second column) as a function of time t (first column) to make a plot like Fig. 1(a). Present your figure in a slide, accompanied by some brief descriptive text on what it shows.

(3) The model. The astronomer asks you to fit the data using a particular model:

$$f(t) = \frac{5}{8\pi}\left(\frac{5GM}{c^3}\right)^{-5/8}(t_0 - t)^{-3/8}. \qquad (1)$$

Here $G = 6.674 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} is Newton's gravitational constant, and $c = 2.998 \times 10^8$ ms^{-1} is the speed of light.

There are two parameters in this model: t_0, the time of when the two bodies merge together, and M, the **chirp mass**. The chirp mass M is commonly expressed as a fraction/multiple of the mass of the Sun ($M_\odot = 1.989 \times 10^{30}$ kg).

Present the formula above on a slide. Briefly describe f, t_0, and M in words on the slide. Briefly describe *why* the frequency is an increasing function of time, referring to the orbit of the compact body.

(4) Fitting a model: theory. On this slide, briefly outline the mathematical theory for how the problem of finding the best-fit parameters β_j for a linear model reduces to the problem of solving the **normal equations**

$$(\mathbf{X}^T\mathbf{X})\boldsymbol{\beta} = \mathbf{X}^T\mathbf{y}, \tag{2}$$

where $\boldsymbol{\beta} = [\beta_0, \beta_1, \ldots, \beta_{m-1}]$ are the fit parameters, $\mathbf{X}$ is an $N \times m$ matrix with rows $[1, t_i^2, t_i^4, \ldots, t_i^{2n}]$ and $\mathbf{y}$ is the data set with N values. (See Lec. 7 and Lab Class 7).

(5) Fitting a model: result.

Write Python code to fit the linear model (1) to the data shown on slide (2). The data file is calibrated so that you can set $t_0 = 0$, to reduce this to a model that is linear in a single parameter β_0 that depends on M. You may find the best-fit value of the chirp mass M by solving the normal equations, or by any other legitimate method.

Present a plot similar to Fig. 1(b). Briefly describe the method used. State the best-fit value of the chirp mass in kilograms. State this value again in units of the solar mass $M_\odot$.

For the final part of the assignment, you have a choice of either 6(a) or 6(b). Option (a) is easier. More marks are available for option (b).

(6a). **The chirp mass** (easier; 1 slide).

The chirp mass M is a particular combination of the masses of the two bodies in the compacy binary, m_1 and m_2 (with $m_1 > m_2$). Look up[2] the formula for M in terms of m_1 and m_2 and present this formula on the slide.

Find m_1 in terms of M in the special case that the two bodies have equal masses ($m_1 = m_2$). Present the result on the slide.

Neutron stars are known to have masses in the range $1.17M_\odot$ – $2.16M_\odot$. Is it plausible that **both** bodies that created the gravitational wave chirp shown on your slide (5) were neutron stars? Briefly explain your reasoning on the slide.

(6b). **Analysing LIGO data** (harder; up to 2 slides). Figure 1(c) and (d) show spectrograms made from real gravitational-wave data collected at the two sites (Hanford and Livingston) of the LIGO experiment.[3]. Both plots show evidence for a 'chirp'; however, the data is noisy, and the chirp feature is quite broad.

The spectrogram data for the Hanford and Livingston detectors is here:

- `http://sam-dolan.staff.shef.ac.uk/mas212/data/spectrogram_H1.csv`
- `http://sam-dolan.staff.shef.ac.uk/mas212/data/spectrogram_L1.csv`

The data is in a tabular format. The times t (in seconds) are given in the first column and the frequencies f (in Hertz) in the first row. The other values in the table give the power in each time and frequency bin.

[2] `https://en.wikipedia.org/wiki/Chirp_mass`

[3] A spectrogram illustrates the power in the data at a particular time and frequency 'bin'

Once again, the aim is to fit the data to the model (1), to estimate the chirp mass M of the system. However, the task is significantly complicated by the fact that the data is now in a pair of 2D tables. Your task is to (i) devise a practical method for estimating M from the data; (ii) implement this in Python code; (iii) describe the steps in your method on the slides. There is no 'right' method that I am looking for; this is a problem-solving exercise. For marking credit, it is important that the steps in your method are described clearly in your slides, and the final estimate of M is presented clearly. You may wish to present a plot showing the agreement between data and model.

The astronomer claims that this chirp could **not** have been created by a pair of neutron stars. Do you agree or disagree with this statement? Briefly explain your reasoning on the slide. (N.B. first read part 6(a), above.)

(optional) **References and sources**. If it was not possible to fit in the references & sources in the other slides, you can add your references and sources in one extra slide.

Guidance:

This assignment will count for $\sim 30\%$ of your module mark, and thus it should be a substantial piece of work. For this assignment, **fair means** include: asking the lecturer for advice; reading online materials; using and adapting short code snippets from lectures; etc. Please avoid verbatim copying, and please cite all sources used. **Unfair means** include (but are not limited to): sharing or distributing files; copying-and-pasting from work that is not your own; posting your work online; passing off other's work as your own; asking for code from last year's cohort; etc.

Note that all submissions will be checked for excessive similarities. Where there is circumstantial evidence of unfair means, I reserve the right to award zero marks for this assignment.