A pulsar is a rapidly-rotating, magnetised neutron star that emits regular pulses of radio waves at rates of up to 1000 pulses per second. Pulsars are highly-accurate cosmic metronomes.

Image: https://commons.wikimedia.org/wiki/File:Chandra-crab.jpg

- The Crab Pulsar (above) revolves 33 times per second. It originated in the supernova of 1054 A.D. seen by Chinese astronomers.

- The first binary pulsar (B1913+16) – a pulsar orbiting a neutron star – was found in 1974.

- The orbital period of the binary pulsar is decreasing at a rate of 76.5\(\mu\)s per year.

- This is because the binary is emitting gravitational waves at exactly the rate predicted by Einstein’s general theory of relativity.
(2) The pulse

(a) Ten pulses from a pulsar

(b) The ‘averaged’ profile

- Plot (a) shows time-series data $y(t)$ from an astronomer’s measurements.
- Plot (b) shows the **averaged pulse**, $\bar{y}(t) = \frac{1}{n} \sum_{k=0}^{n-1} y(t + kT)$

where $T = 1.234$ is the period and $n = 10$. 
(3) A linear model: theory

- To fit a model $f(t; \beta_j)$ to the data, we seek the parameters $\beta_j$ that minimize the sum of squared residuals
  \[
  S(\beta_j) = \sum_{k=1}^{N} (y(t_k) - f(t_k; \beta_j))^2.
  \]

- A linear model can be written $f(t; \beta_j) = \sum_{j=0}^{m-1} \beta_j \phi_j(t)$, where $\phi_j(t)$ are functions. For an even polynomial, $\phi_j(t) = t^{2j}$.

- Setting the partial derivatives $\frac{\partial S}{\partial \beta_j}$ to zero yields the normal equations
  \[
  (X^T X) \beta = X^T y,
  \]

- Here $X$ is an $N \times m$ matrix with rows $[\phi_0(t_i), \phi_1(t_i), \ldots, \phi_{m-1}(t_i)]$ and $y$ is the averaged pulse data with $N$ values.

- The normal equations (a linear system) are solved by e.g. Gaussian elimination to obtain the best-fit parameters $\beta$. 
I used `np.linalg.solve()` to solve the normal equations to fit an even polynomial of highest power $t^{2(m-1)}$ with $m = 10$.

The best-fit polynomial is

$$f(t) \approx 0.8197 - 78.22t^2 + 3118.0t^4 - \ldots$$

A high-order polynomial is needed to fit the main pulse, but the high order creates spurious oscillations toward the edges of the domain.
Next, I fitted a Gaussian model with four parameters \( \{c, A, \tau, \sigma\} \) using `scipy.optimize.curve_fit()`. 

\[
f(t; \beta_j) = c + A \exp\left(-\frac{(t - \tau)^2}{2\sigma^2}\right).
\]

The best-fit parameters were:

\[c = 0.14302, \quad A = 0.80902, \quad \tau = -0.00419, \quad \text{and} \quad \sigma = 0.04623.\]
(6) Which model is best?

- The Gaussian model is superior to the polynomial model, as it achieves a smaller Root Mean Square Deviation (RMSD) with fewer parameters.

<table>
<thead>
<tr>
<th>parameters</th>
<th>linear model</th>
<th>non-linear model</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSD</td>
<td>0.0691</td>
<td>0.0513</td>
</tr>
</tbody>
</table>

- Neither model fits all the structure in the data. There is correlation between neighbouring residuals:
The European Pulsar Network database http://www.epta.eu.org/epndb/ hosts data on real pulsars observed at various frequencies across the electromagnetic spectrum.

I tried fitting a double-Gaussian model with 7 parameters

\[ f(t; \beta_j) = c + A_1 \exp\left(-\frac{(t - \tau_1)^2}{2\sigma_1^2}\right) + A_2 \exp\left(-\frac{(t - \tau_2)^2}{2\sigma_2^2}\right) \]

to three data sets:

(a) the Binary Pulsar (PSR B1913+16) at 1560MHz

http://www.epta.eu.org/epndb/#kxl+98/J1915+1606/kxl+98.epn

(b) the Crab pulsar (PSR B0531+21) at 925MHz

http://www.epta.eu.org/epndb/#gl98/J0534+2200/gl98_925.epn

(c) the Crab pulsar at 4885MHz.

http://www.epta.eu.org/epndb/#mh99/J0534+2200/mh99_4885.epn
The EPN database: an investigation

(a) Binary pulsar PSR B1913+16 at 1560MHz

(b) Crab pulsar PSR B0531+21 at 925MHz

(c) Crab pulsar PSR B0531+21 at 4885MHz

- The double-Gaussian model fits very well in case (b) & quite well in case (a).
- It is not suitable in case (c), which has four distinct features.