MAS212 Assignment 3: Pulsars

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(1) Introduction

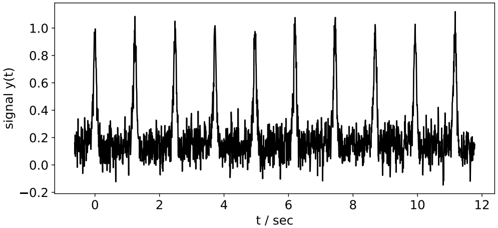
A **pulsar** is a rapidly-rotating, magnetised **neutron star** that emits regular pulses of radio waves at rates of up to 1000 pulses per second.

Pulsars are highly-accurate cosmic metronomes.

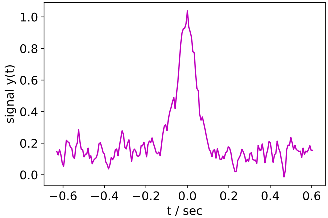
Image: https://commons.wikimedia.org/wiki/File:Chandra-crab.jpg

- The **Crab Pulsar** (above) revolves 33 times per second. It originated in the supernova of 1054 A.D. seen by Chinese astronomers.
- The first **binary pulsar** (B1913+16) – a pulsar orbiting a neutron star – was found in 1974.
- The orbital period of the binary pulsar is decreasing at a rate of $76.5\mu s$ per year.
- This is because the binary is emitting **gravitational waves** at exactly the rate predicted by Einstein's general theory of relativity.

(2) The pulse



(a) Ten pulses from a pulsar



(b) The 'averaged' profile

- Plot (a) shows time-series data $y(t)$ from an astronomer's measurements
- Plot (b) shows the **averaged pulse**,

$$\bar{y}(t) = \frac{1}{n} \sum_{k=0}^{n-1} y(t + kT)$$

where $T = 1.234$ is the period and $n = 10$.

(3) A linear model: theory

- To fit a **model** $f(t;\beta_j)$ to the data, we seek the **parameters** β_j that minimize the **sum of squared residuals**

$$S(\beta_j) = \sum_{k=1}^{N} \left(y(t_k) - f(t_k;\beta_j)\right)^2 .$$

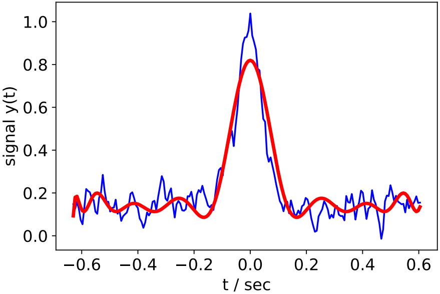
- A **linear model** can be written $f(t;\beta_j) = \sum_{j=0}^{m-1} \beta_j \phi_j(t)$, where $\phi_j(t)$ are functions. For an **even** polynomial, $\phi_j(t) = t^{2j}$.
- Setting the partial derivatives $\frac{\partial S}{\partial \beta_j}$ to zero yields the **normal equations**

$$(\mathbf{X}^T\mathbf{X})\boldsymbol{\beta} = \mathbf{X}^T\mathbf{y},$$

- Here $\mathbf{X}$ is an $N \times m$ matrix with rows $[\phi_0(t_i), \phi_1(t_i), \ldots, \phi_{m-1}(t_i)]$ and $\mathbf{y}$ is the averaged pulse data with N values.
- The normal equations (a linear system) are solved by e.g. Gaussian elimination to obtain the best-fit parameters $\boldsymbol{\beta}$.

(4) A linear model: result

- I used `np.linalg.solve()` to solve the normal equations to fit an even polynomial of highest power $t^{2(m-1)}$ with $m = 10$.



- The best-fit polynomial is

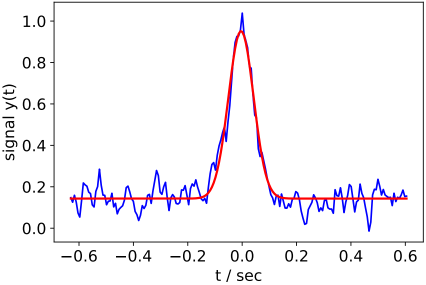
$$f(t) \approx 0.8197 - 78.22t^2 + 3118.0t^4 - \dots$$

- A high-order polynomial is needed to fit the main pulse, but the high order creates spurious oscillations toward the edges of the domain.

(5) A non-linear model

- Next, I fitted a Gaussian model with four parameters $\{c, A, \tau, \sigma\}$ using `scipy.optimize.curve_fit()`

$$f(t;\beta_j) = c + A\exp\left(-\frac{(t-\tau)^2}{2\sigma^2}\right).$$



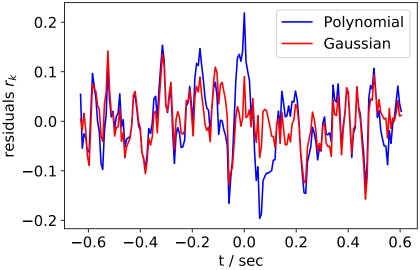
- The best-fit parameters were:
$c = 0.14302$, $A = 0.80902$, $\tau = -0.00419$, and $\sigma = 0.04623$.

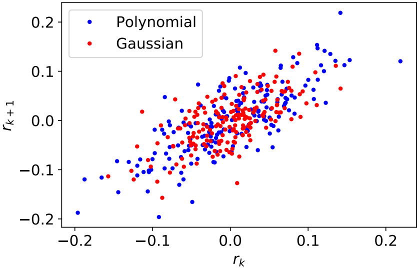
(6) Which model is best?

- The Gaussian model is superior to the polynomial model, as it achieves a smaller Root Mean Square Deviation (RMSD) with fewer parameters.

	linear model	non-linear model
parameters	10	4
RMSD	0.0691	0.0513

- Neither model fits all the structure in the data. There is correlation between neighbouring residuals:





(7) The EPN database: an investigation

- The European Pulsar Network database `http://www.epta.eu.org/epndb/` hosts data on real pulsars observed at various frequencies across the electromagnetic spectrum.
- I tried fitting a double-Gaussian model with 7 parameters

$$f(t;\beta_j) = c + A_1 \exp\left(-\frac{(t-\tau_1)^2}{2\sigma_1^2}\right) + A_2 \exp\left(-\frac{(t-\tau_2)^2}{2\sigma_2^2}\right)$$

to three data sets:

(a) the Binary Pulsar (PSR B1913+16) at 1560MHz

`http://www.epta.eu.org/epndb/#kxl+98/J1915+1606/kxl+98.epn`

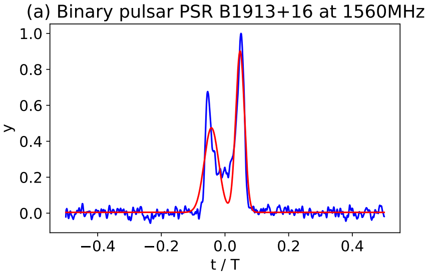
(b) the Crab pulsar (PSR B0531+21) at 925MHz

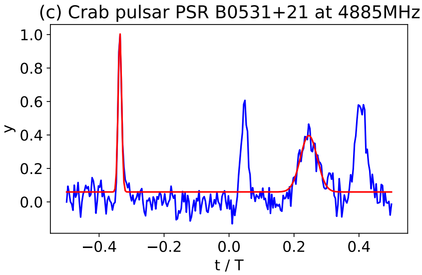
`http://www.epta.eu.org/epndb/#gl98/J0534+2200/gl98_925.epn`

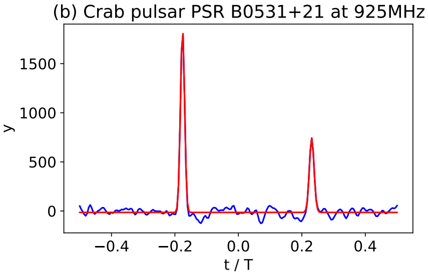
(c) the Crab pulsar at 4885MHz.

`http://www.epta.eu.org/epndb/#mh99/J0534+2200/mh99_4885.epn`

(8) The EPN database: an investigation







- The double-Gaussian model fits very well in case (b) & quite well in case (a).
- It is not suitable in case (c), which has **four** distinct features.