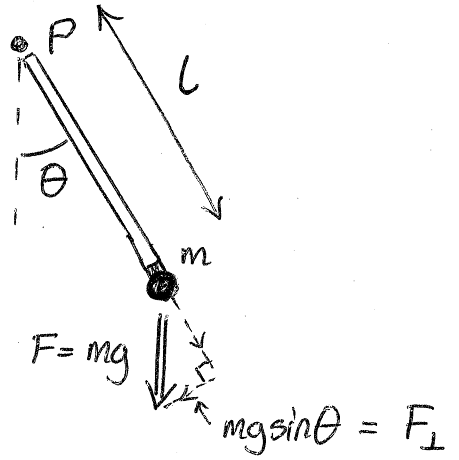
Assignment #2 Background: A damped rigid pendulum

Consider a massless rod of length l attached to a pivot P at one end, and with a mass m at the other.
Assume that it can swing freely under gravity, but with a frictional force that is proportional to its speed.



The moment of inertia I is
$$I = ml^2.$$

Let θ be the angle to the vertical. The angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$. The speed of the mass is $l\dot{\theta}$.
(Assume that the rod can "swing over the top" and $\theta \in (-\pi, +\pi]$.)

The mass is subject to a "restoring torque" $T_g = -mgl\sin\theta$ due to gravity.

There is also a frictional torque of $T_f = -\mu l \dot{\theta}$ where μ is some constant of friction.

The equation of motion is "torque = moment of inertia × angular acceleration" (this is the angular version of $F = ma$).

$$\Rightarrow \quad T_f + T_g = I\ddot{\theta} = ml^2\ddot{\theta}$$

dividing by ml^2

$$\Rightarrow \quad \ddot{\theta} = -\frac{g}{l}\sin\theta - \frac{\mu}{l}\dot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} + \frac{\mu}{ml}\dot{\theta} + \frac{g}{l}\sin\theta = 0$$

This is the equation for a damped <u>undriven</u> rigid pendulum.

In this assignment we will consider the non-linear 2nd order ODE:

$$\ddot{\theta} + \underbrace{2\gamma}_{\frac{\mu}{\cancel{ml}}}\dot{\theta} + \underbrace{\omega^2}_{g/l}\sin\theta = f(t)$$

← driving "force"