MAS212 Scientific Computing and Simulation

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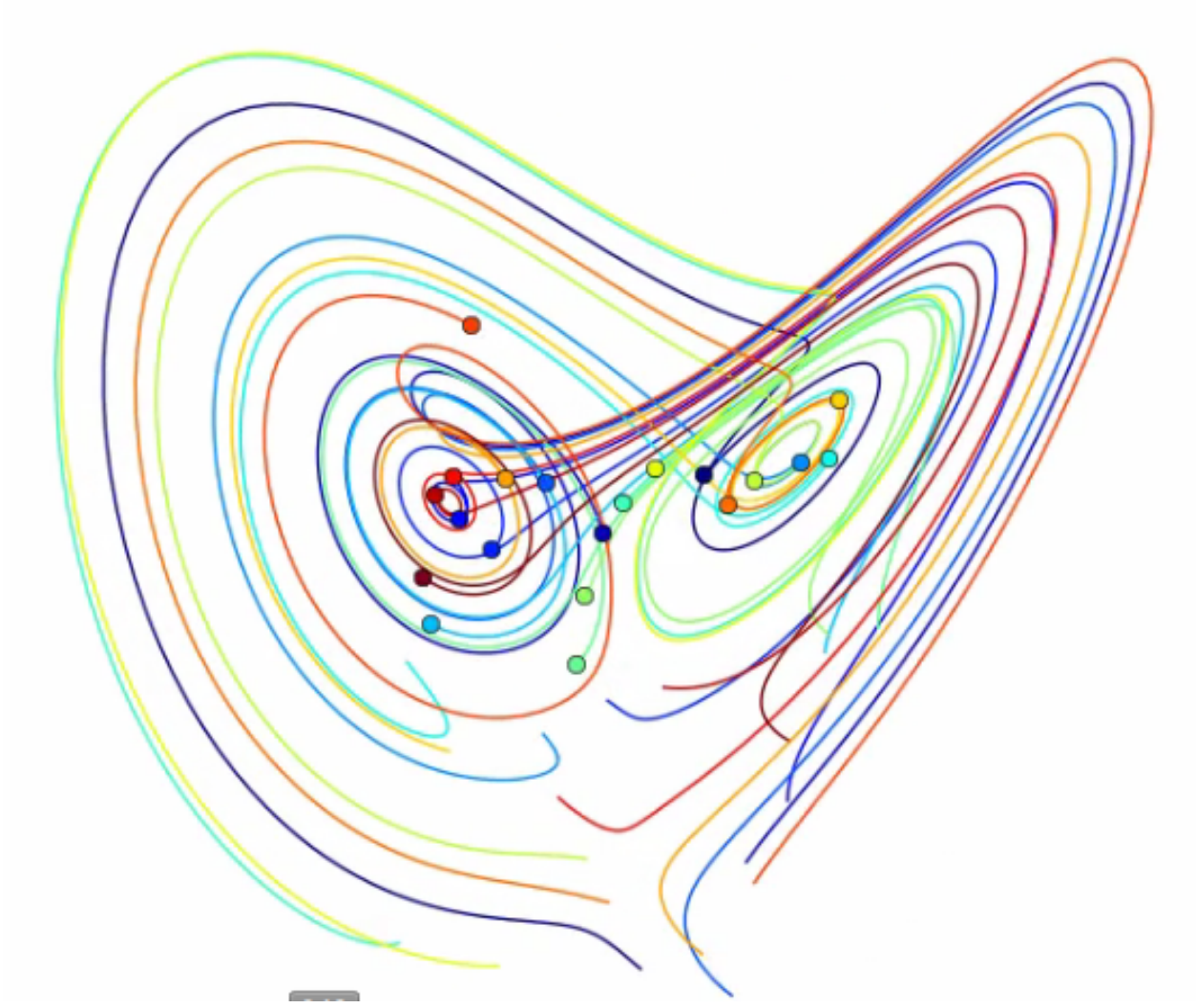


https://jakevdp.github.io/blog/2013/02/16/animating-the-lorentz-system-in-3d/

Today's lecture

- Animation using `matplotlib.animate.FuncAnimation()`
- Examples:
  - A moving sine wave
  - The logistic map
  - Van der Pol oscillator
  - 3D animation: the Lorenz equations
  - Particles in a box
- Code on course website:
  `http://sam-dolan.staff.shef.ac.uk/mas212/code`

A note on Python syntax

```
>>> (1,2)  # a tuple
(1, 2)
>>> 1,2  # If I omit the brackets I still get a tuple
(1, 2)
>>> # How do I write a tuple with just one element?
>>> (1) # This doesn't work ...
1
>>> (1,) # This is how to write a tuple with just one element
(1, )
>>> # Functions often return lists or tuples
>>> # For example, plot() returns a list of Line2D objects.
>>> x = np.linspace(0, 4, 100)
>>> l1  = plt.plot(x, np.sin(x))  # compare these lines
>>> l2, = plt.plot(x, np.sin(x))   # and note the comma
>>> type(l1)   # l1 is the list
<class 'list'>
>>> type(l2)   # l2 is the first element of the list
<class 'matplotlib.lines.Line2D'>
```

Animation

Why? Animations can be used to . . .

- . . . help us understand systems that evolve in time
- . . . understand solutions of ODEs
- . . . impress people! (use in moderation)

Animation

What will we use?

- The `matplotlib.animation` module
- The `FuncAnimation()` function
- Code examples here: `http://matplotlib.org/examples/animation/`
- A good tutorial here: `https://jakevdp.github.io/blog/2012/08/18/matplotlib-animation-tutorial/`

```
import numpy as np
from matplotlib import pyplot as plt
from matplotlib import animation
fig = plt.figure()
ax = plt.axes(xlim=(0, 2), ylim=(-2, 2))
line, = ax.plot([], [], lw=2)  # an empty line
def init():  # Initialize with a blank plot
    line.set_data([], [])   # line is a global variable
    return line,
def animate(i):  # Plot a moving sine wave
    x = np.linspace(0, 2, 1000)
    y = np.sin(2 * np.pi * (x - 0.01 * i))
    line.set_data(x, y)
    return line,
anim = animation.FuncAnimation(fig, animate, init_func=init,
                               frames=200, interval=20, blit=True)
plt.show()
```

FuncAnimation : step-by-step

- Import the necessary modules:

```
import numpy as np
from matplotlib import pyplot as plt
from matplotlib import animation
```

- Set up a `Figure`, an `AxesSubplot` (blank plot), and an **empty line** which can be changed

```
fig = plt.figure()
ax = plt.axes(xlim=(0, 2), ylim=(-2, 2))
line, = ax.plot([], [], lw=2)   # an empty line
```

- Here `line` is a `Line2D` object.
  This is the only element which will change during the animation; the rest of the plot stays the same.

`FuncAnimation` : step-by-step

- The initialization function `init()` is called before the animation begins
- We set the line data to empty lists: no line will be drawn

```
def init():
    line.set_data([], [])   # line is a global variable
    return line,
```

- This function must return the `Line2D` object

FuncAnimation : step-by-step

- This function makes a new frame:

```
def animate(i):  # Plot a moving sine wave
    x = np.linspace(0, 2, 1000)
    y = np.sin(2 * np.pi * (x - 0.01 * i))
    line.set_data(x, y)
    return line,
```

- This function is called repeatedly, with `i` incremented, each time a new frame is to be drawn.
- The function should return a tuple of the elements in the plot which have been changed. These elements will then be re-drawn.

FuncAnimation : step-by-step

- Now we create the FuncAnimation object, passing the figure, the animate function and the init function

```
anim = animation.FuncAnimation(fig, animate,
        init_func=init, frames=200, interval=20, blit=True)
plt.show()
```

- We also set the total number of frames to 200, and the delay between frames to 20 milliseconds
- The FuncAnimation object must persist, so we assign it to a variable (anim)
- Setting blit to True ensures that only those elements of the plot that have changed will be redrawn. This makes the animation fast & smooth.

‘Blitting’

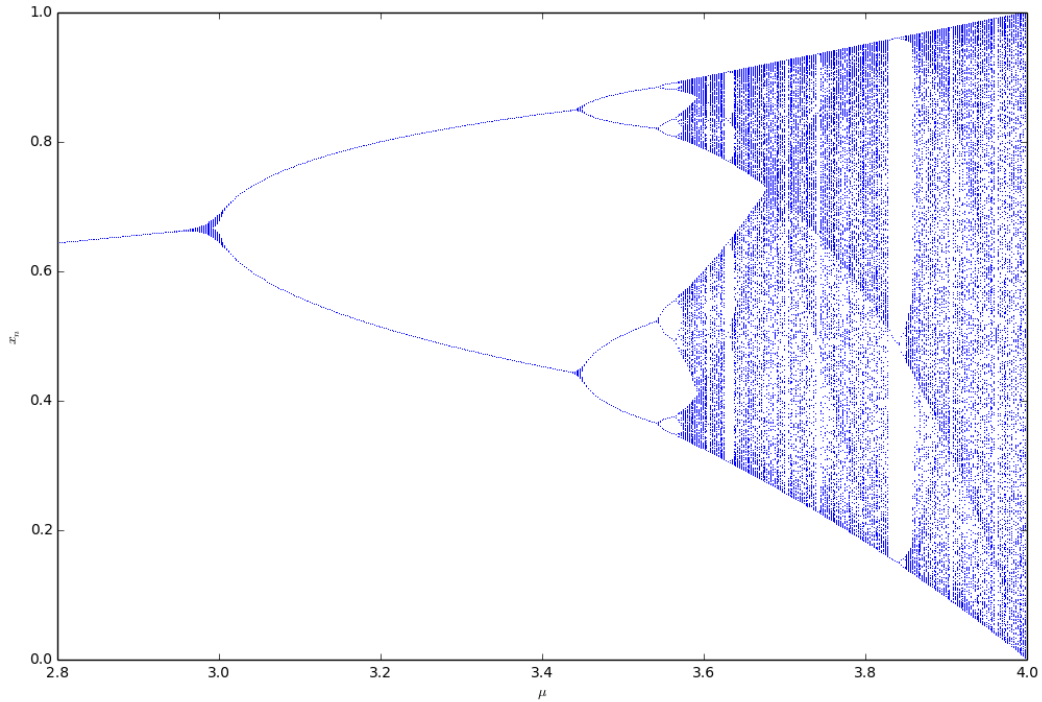
Bit blit (bit-boundary block transfer) is a computer graphics operation in which several bitmaps are combined into one using a raster operator.

- `http://en.wikipedia.org/wiki/Bit_blit`
- e.g., if a line changes, but the background axes stay the same, we don’t want to redraw the entire image.
- ‘Blitting’ optimizes drawing
  $\Rightarrow$ animations are smoother & faster.

Animating the logistic map

$$x \to \mu x(1 - x)$$

The bifurcation diagram for the logistic map looks like this:



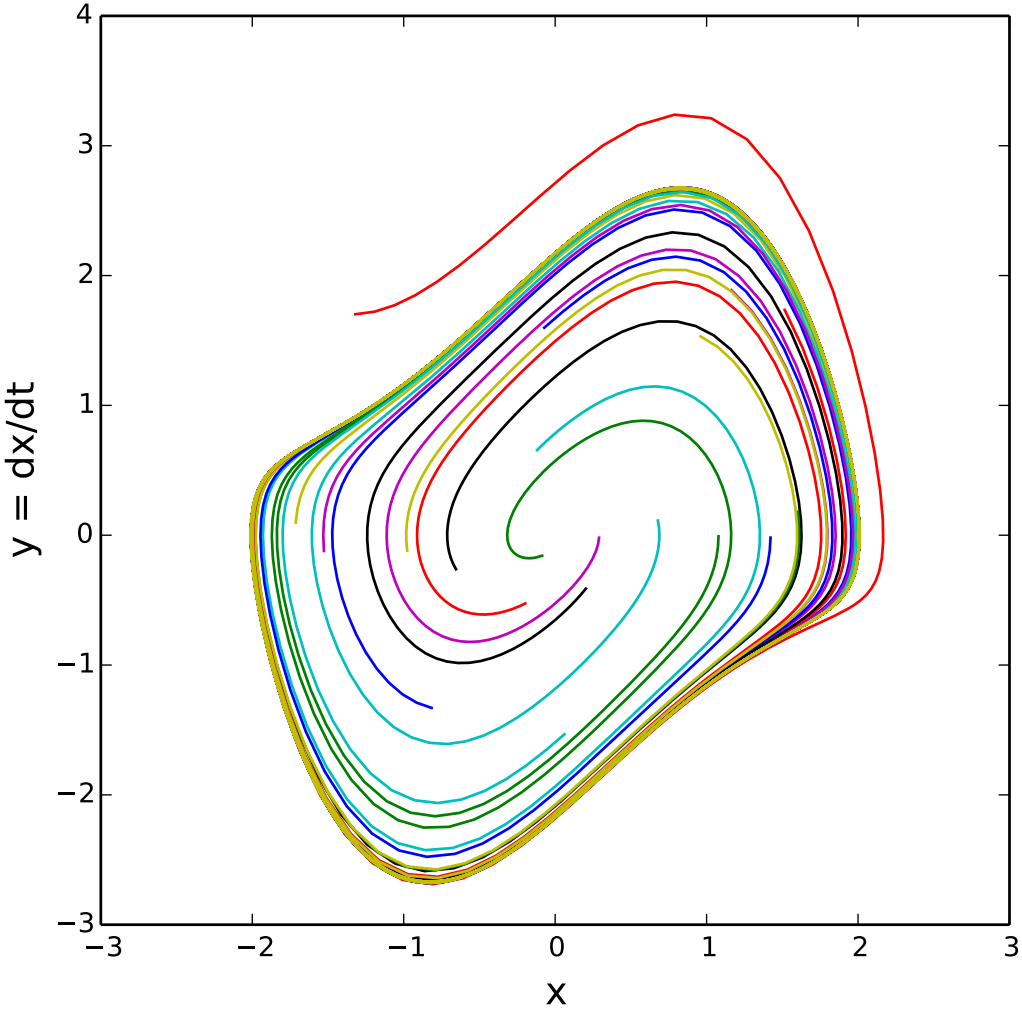
Let's increase the parameter μ slowly . . .

Animating the logistic map

```
fig = plt.figure()
ax = plt.axes(xlim=(0,50), ylim=(0.0, 1.0))
line, = ax.plot([], [], 'ro')
def logisticmap(mu=2.0, N=100):
    xs = 0.5*np.ones(N)
    for i in np.arange(N-1):
        xs[i+1] = mu*xs[i]*(1.0-xs[i])
    return xs
def init():
    line.set_data([],[])
    return line,
def animate(i):
    data = logisticmap(2.0+i*0.01)[50:]
    line.set_data(np.arange(len(data)), data)
    return line,
anim = animation.FuncAnimation(fig, animate, init_func=init,
          frames=200, interval=100, blit=True)
plt.show()
```

Animating the van der Pol oscillator

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$$



Animating the van der Pol oscillator

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$$

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \mu(1 - x^2)y - x \end{aligned}$$

First, solve the ODEs to obtain data:

```
mu = 1.0   # parameter value
def dx_dt(x, t):
    return [x[1], mu*(1-x[0]**2)*x[1] - x[0]]
def random_ic(scalefac=2.0):  # generate initial condition
    return scalefac*(2.0*np.random.rand(2) - 1.0)
ts = np.linspace(0.0, 40.0, 400)
nlines = 20
linedata = []
for ic in [random_ic() for i in range(nlines)]:
    linedata.append( odeint(dx_dt, ic, ts) )
```

Animating the van der Pol oscillator

Now create the animation:

```
fig = plt.figure()
ax = plt.axes(xlim=(-3,3), ylim=(-3, 3))
line, = ax.plot([], [], 'ro')
npts = len(linedata[0][:,0])
def init():
    line.set_data([],[])
    return line,
def animate(i):
    line.set_data([l[i,0] for l in linedata],
                  [l[i,1] for l in linedata])
    return line,
anim = animation.FuncAnimation(fig, animate, init_func=init,
       frames=npts, interval=50, blit=True)
plt.show()
```

The Lorenz equations

In 1963, Edward Lorenz developed a simplified mathematical model for atmospheric convection. The model is a system of three ordinary differential equations now known as the Lorenz equations:

$$\begin{aligned} \frac{dx}{dt} &= \sigma(y - x) \\ \frac{dy}{dt} &= x(\rho - z) - y \\ \frac{dz}{dt} &= xy - \beta z \end{aligned}$$

- σ, ρ, β are parameters. Set e.g. $\sigma = 10$, $\beta = 8/3$, $\rho = 28$.
- **Non-linear** because of xy term
- Displays deterministic chaos
- 3 dimensional system with a **strange attractor**

Animating the Lorenz equations

- Code for animating 3D Lorenz equations on Jake Vanderplas' Python blog:
  `https://jakevdp.github.io/blog/2013/02/16/animating-the-lorentz-system-in-3d/`